Add to PS3.3, Chapter C.20.2.1.1

C.20.2.1.1 Frame of Reference Transformation Matrix

The Frame of Reference Transformation Matrix (3006,00C6) $^\text{A}M_\text{B}$ describes how to transform a point $(^\text{A}x,^\text{A}y,^\text{A}z)$ with respect to RCS$^\text{B}$ into $(^\text{A}x,^\text{A}y,^\text{A}z)$ with respect to RCS$^\text{A}$ according to the equation below.

$$
\begin{bmatrix}
^\text{A}x \\
^\text{A}y \\
^\text{A}z \\
1
\end{bmatrix} =
\begin{bmatrix}
M_{11} & M_{12} & M_{13} & T_x \\
M_{21} & M_{22} & M_{23} & T_y \\
M_{31} & M_{32} & M_{33} & T_z \\
0 & 0 & 0 & 1
\end{bmatrix}
\begin{bmatrix}
^\text{B}x \\
^\text{B}y \\
^\text{B}z \\
1
\end{bmatrix}
$$

The Matrix Registration is expressible as multiple matrices, each in a separate item of the Matrix Sequence (0070,030A). The equation below specifies the order of the matrix multiplication where $M_1$, $M_2$, and $M_3$ are the first, second and third items in the sequence.

$$
[x' \ y' \ z' \ 1]^T = M_3(M_2(M_1[x \ y \ z \ 1]^T))
$$
where \[
\begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}^T = \begin{bmatrix}
x \\
y \\
z \\
1
\end{bmatrix}
\]

Registration often involves two or more RCS, each with a corresponding Frame of Reference Transformation Matrix. For example, another Frame of Reference Transformation Matrix \( ^A M_C \) can describe how to transform a point \((^C x, ^C y, ^C z)\) with respect to RCS \( C \) into \((^A x, ^A y, ^A z)\) with respect to RCS \( A \). It is straightforward to find the Frame of Reference Transformation Matrix \( ^B M_C \) that describes how to transform the point \((^C x, ^C y, ^C z)\) with respect to RCS \( C \) into the point \((^B x, ^B y, ^B z)\) with respect to RCS \( B \). The solution is to invert \( ^A M_B \) and multiply by \( ^A M_C \), as shown below:

\[
\begin{bmatrix}
^B x \\
^B y \\
^B z \\
1
\end{bmatrix} = (^A M_B)^{-1} \ast ^A M_C \begin{bmatrix}
^C x \\
^C y \\
^C z \\
1
\end{bmatrix}
\]

If two or more transformation matrices describe the relation between Patient coordinates and a device-centric Well-known Frame of Reference, any calculations assuming transitivity via the Well-known Frame of Reference must be performed with great care to assure that both registrations reflect the same positioning of the patient with respect to the common Well-known Frame of Reference.