DICOM Correction Proposal

STATUS	Draft Final Text
Date of Last Update	2012/11/04
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Correction Number CP-1213

Log Summary: Clarify affine transformation matrix constraints

Name of Standard

PS 3.3, 3.17 2011

Rationale for Correction:

The explanatory text in PS 3.17 Annex P makes it clear that the transformation matrix uses homogeneous coordinates, and the normative text in the description of the attribute uses the word “homogeneous”, but the normative text for the AFFINE type of matrix contradicts this stating that that the element values are “unconstrained”. Correct the normative text.

Also, the question arises as to whether other values may be used in the matrix when the type (which is a defined term not an enumerated value) is not one of the specified types. Make these enumerated values.

Also, the same matrix is historically used in the RT Structure Set but with a different type attribute with a single defined term of HOMOGENEOUS; add a note about this too. It is not clear whether in the RT use case this should be an enumerated value, or whether the matrix should be constrained to be homogeneous too.

The Image - Equipment Coordinate Relationship Module and the Ultrasound Frame of Reference Module are also updated correspondingly, to replace the use of the term “homogeneous” with “rigid”, since the normative text accompanying both defines a rigid rather than less constrained affine transformation.

Correction Wording:

Amend PS 3.3 C.20.2:

C.20.2 Spatial Registration Module

...

Table C.20.2-1
SPATIAL REGISTRATION MODULE ATTRIBUTES

Attribute Name	Tag	Type	Attribute Description
>>Matrix Sequence	(0070,030A)	1	Specifies one transformation~~,~~ that registers the Source RCS/images to the Registered RCS. It is expressible as multiple matrices, each in a separate item of the sequence. One or more Items shall be included in this sequence. The item order is significant and corresponds to matrix multiplication order. See C.20.2.1.1.

>>>Frame of Reference Transformation Matrix	(3006,00C6)	1	A 4x4 ~~**homogeneous**~~ **affine** transformation matrix that registers a **homogeneous** coordinate system A to B. Matrix elements shall be listed in row-major order. See C.20.2.1.1.
>>>Frame of Reference Transformation Matrix Type	(0070,030C)	1	Type of Frame of Reference Transformation Matrix (3006,00C6). ~~**Defined terms**~~ **Enumerated Values**: RIGID RIGID_SCALE AFFINE See C.20.2.1.2
…	…	…	…

C.20.2.1 Registration Module Attribute Descriptions

C.20.2.1.1 Frame of Reference Transformation Matrix

The Frame of Reference Transformation Matrix (3006,00C6) ${}^{A}M_{B}$ describes how to transform a point $({}^{B}x, {}^{B}y, {}^{B}z)$ with respect to RCS_B into $({}^{A}x, {}^{A}y, {}^{A}z)$ with respect to RCS_A according to the equation below.

$$\begin{bmatrix} {}^{A}x \\ {}^{A}y \\ {}^{A}z \\ 1 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & T_x \\ M_{21} & M_{22} & M_{23} & T_y \\ M_{31} & M_{32} & M_{33} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}x \\ {}^{B}y \\ {}^{B}z \\ 1 \end{bmatrix}$$

The Matrix Registration is expressible as multiple matrices, each in a separate item of the Matrix Sequence (0070,030A). The equation below specifies the order of the matrix multiplication where $\mathbf{M_1}$, $\mathbf{M_2}$ and $\mathbf{M_3}$ are the first, second and third items in the sequence.

$$\begin{bmatrix} x' & y' & z' & 1 \end{bmatrix}^T = \mathbf{M}_3\left(\mathbf{M}_2\left(\mathbf{M}_1\begin{bmatrix} x & y & z & 1 \end{bmatrix}^T\right)\right)$$

$$\text{where } \begin{bmatrix} x & y & z & 1 \end{bmatrix}^T = \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Registration often involves two or more RCS, each with a corresponding Frame of Reference Transformation Matrix. For example, another Frame of Reference Transformation Matrix ${}^{A}M_{C}$ can describe how to transform a point $({}^{C}x, {}^{C}y, {}^{C}z)$ with respect to RCS_C into $({}^{A}x, {}^{A}y, {}^{A}z)$ with respect to RCS_A. It is straightforward to find the Frame of Reference Transformation Matrix ${}^{B}M_{C}$ that describes how to transform the point $({}^{C}x, {}^{C}y, {}^{C}z)$ with respect to RCS_C into the point $({}^{B}x, {}^{B}y, {}^{B}z)$ with respect to RCS_B. The solution is to invert ${}^{A}M_{B}$ and multiply by ${}^{A}M_{C}$, as shown below:

$$\begin{bmatrix} {}^{B}x \\ {}^{B}y \\ {}^{B}z \\ 1 \end{bmatrix} = ({}^{A}\mathbf{M}_{B})^{-1} * {}^{A}\mathbf{M}_{C} \begin{bmatrix} {}^{C}x \\ {}^{C}y \\ {}^{C}z \\ 1 \end{bmatrix}$$

C.20.2.1.2 Frame of Reference Transformation Matrix Type

There are three types of Registration Matrices:

RIGID: This is a registration involving only translations and rotations. Mathematically, the matrix is constrained to be orthonormal and describes six degrees of freedom: three translations, and three rotations.

RIGID_SCALE: This is a registration involving only translations, rotations and scaling. Mathematically, the matrix is constrained to be orthogonal and describes nine degrees of freedom: three translations, three rotations and three scales. This type of transformation is sometimes used in atlas mapping.

AFFINE: This is a registration involving translations, rotations, scaling and shearing. Mathematically, there are no constraints on the elements of the Frame of Reference Transformation Matrix **other than that the last row shall be (0,0,0,1) to preserve the homogeneous coordinates**, so it conveys twelve degrees of freedom. This type of transformation is sometimes used in atlas mapping.

> **Note: The AFFINE value for Frame of Reference Transformation Matrix Type (0070,030C) has the same meaning as the use of the HOMOGENEOUS value for Frame of Reference Transformation Type (3006,00C4) in the Structure Set module. See Section C.8.8.5.**

See the PS 3.17 Annex on Transforms and Mappings for more detail.

Amend PS 3.3 C.8.8.5:

C.8.8.5 Structure Set Module

A structure set defines a set of areas of significance. Each area can be associated with a Frame of Reference and zero or more images. Information that can be transferred with each region of interest (ROI) includes geometrical and display parameters, and generation technique.

Table C.8-41—STRUCTURE SET MODULE ATTRIBUTES

Attribute Name	Tag	Type	Attribute Description
…	…	…	…
>>Frame of Reference Transformation Type	(3006,00C4)	1	Type of Transformation. Defined Terms: HOMOGENEOUS
>>Frame of Reference Transformation Matrix	(3006,00C6)	1	Four-by-four transformation Matrix from Related Frame of Reference to current Frame of Reference. Matrix elements shall be listed in row-major order. See C.8.8.5.2.
…	…	…	…

…

C.8.8.5.2 Frame of Reference Transformation Matrix

In a rigid body system, two coordinate systems can be related using a single 4 x 4 transformation matrix to describe any rotations and/or translations necessary to transform coordinates from the related coordinate system (frame of reference) to the primary system. The equation performing the transform from a point (X',Y',Z') in the related coordinate system to a point (X,Y,Z) in the current coordinate system can be shown as follows, where for **affine** ~~homogeneous~~ transforms **of homogeneous coordinates** $M_{41} = M_{42} = M_{43} = 0$ and $M_{44} = 1$:

$$\begin{matrix} X \\ Y \\ Z \end{matrix} \quad \begin{matrix} \\ = \\ \end{matrix} \quad \begin{matrix} M_{11} & M_{12} & M_{13} & M_{14} \\ M_{21} & M_{22} & M_{23} & M_{24} \\ M_{31} & M_{32} & M_{33} & M_{34} \end{matrix} \quad \begin{matrix} \\ \times \\ \end{matrix} \quad \begin{matrix} X' \\ Y' \\ Z' \end{matrix}$$

1 M_{41} M_{42} M_{43} M_{44} 1

Note: The HOMOGENEOUS value for Frame of Reference Transformation Type (3006,00C4) has the same meaning as the use of the AFFINE value for Frame of Reference Transformation Matrix Type (0070,030C) in the Spatial Registration module. See Section C.20.2.1.2.

Amend PS 3.3 C.7.6.21:

C.7.6.21 Image - Equipment Coordinate Relationship Module

This section describes the Image - Equipment Coordinate Relationship module. Table C.7.6.21-1 contains the attributes that specify how the equipment (e.g. gantry) and patient oriented coordinate system (in conjunction with the Image Position (Patient) (0020,0032) and Image Orientation (Patient) (0020,0037) attributes) are related.

Table C.7.6.21-1
IMAGE - EQUIPMENT COORDINATE RELATIONSHIP MODULE ATTRIBUTES

Attribute Name	Tag	Type	Attribute Description
Image to Equipment Mapping Matrix	(0028,9520)	1	A 4x4 ~~**homogeneous**~~ **rigid** transformation matrix that maps patient coordinate space of the reconstructed image to the equipment defined original coordinate space. Matrix elements shall be listed in row-major order. See C.7.6.21.1.
…	…	…	…

C.7.6.21.1 Image to Equipment Mapping Matrix

The Image to Equipment Mapping Matrix (0028,9520) is used to describe the relationship between the Patient oriented coordinate system and a modality specific equipment coordinate system. This mapping can only be used with systems that have a well-defined equipment coordinate system (such as XA, etc.).

The Image to Equipment Mapping Matrix ${}^{A}M_{B}$ describes how to transform a point $({}^{B}x, {}^{B}y, {}^{B}z)$ with respect to the Patient coordinate system into $({}^{A}x, {}^{A}y, {}^{A}z)$ with respect to the equipment coordinate system according to the equation below.

$$\begin{bmatrix} {}^{A}x \\ {}^{A}y \\ {}^{A}z \\ 1 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & T_x \\ M_{21} & M_{22} & M_{23} & T_y \\ M_{31} & M_{32} & M_{33} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}x \\ {}^{B}y \\ {}^{B}z \\ 1 \end{bmatrix}$$

The Image to Equipment Mapping Matrix is a rigid transformation that involves only translations and rotations. Mathematically, the matrix shall be orthonormal and can describe six degrees of freedom: three translations, and three rotations.

Note: Both the Patient Coordinate System and the Equipment Coordinate System are expressed in millimeters.

Amend PS 3.3 C.8.24.2:

C.8.24.2 Ultrasound Frame of Reference Module

Table C.8.24.2-1 specifies the attributes of the Ultrasound Frame Of Reference Module. See C.8.24.2.1 for an overview of the Ultrasound Frame Of Reference Module.

Table C.8.24.2-1
ULTRASOUND FRAME OF REFERENCE MODULE ATTRIBUTES

Attribute Name	Tag	Type	Attribute Description
…	…	…	…
Volume to Transducer Mapping Matrix	(0020,9309)	1	A 4x4 ~~**homogeneous**~~ **rigid** transformation matrix that maps the Volume Frame of Reference **homogeneous** coordinate system (X_V, Y_V, Z_V) to the Transducer Frame of Reference **homogeneous** coordinate system (X_X,Y_X, Z_X). Matrix elements shall be listed in row-major order. See Section C.8.24.2.1 for details.
…	…	…	…
Volume to Table Mapping Matrix	(0020,930A)	1C	A 4x4 ~~**homogeneous**~~ **rigid** transformation matrix that maps the Volume Frame of Reference **homogeneous** coordinate system (X_V,Y_V, Z_V) to the Table Frame of Reference **homogeneous** coordinate system (X_T,Y_T, Z_T). Matrix elements shall be listed in row-major order. See Section C.8.24.2.2 for details. Required if Patient Frame of Reference Source (0020,930C) is TABLE.

C.8.24.2.1 Ultrasound Frame of Reference Module Overview

The Ultrasound Frame of Reference Module is used to relate the image planes to a frame of reference appropriate for the ultrasound modality, most notably a volume-based frame of reference. There are many different transducer scan acquisition geometries used in 3D ultrasound imaging. Regardless of the acquisition geometry, after acquisition of the initial scan images comprising the volume, the ultrasound (US) scanner will assemble (reformat) the data into a proper Cartesian volume with the assumption that the data are related through a Right-Hand Coordinate System (RHCS). x-positions are defined in mm with positive values increasing towards the right. y-positions are defined in mm with positive values in the direction of increasing image depth. z-positions are defined in mm with positive values in the direction as defined in a right-hand coordinate system.

A Cartesian volume will consist of a series of 1 to n parallel planes. The image planes comprising the Cartesian volume are typically oriented during creation of the volume so that the best image quality is in the XY plane. Table C.8.24.2-1 specifies the attributes of the Ultrasound Frame of Reference Module. There are three levels of detail for the Ultrasound Frame of Reference: Volume, Transducer and Table.

C.8.24.2.1.1 Volume Frame of Reference

The Volume Frame of Reference is a Right-hand Coordinate System consisting of a Volume Origin at the location (0,0,0) and mutually orthogonal X_V, Y_V, and Z_V axes in a Right-Hand Coordinate System. The particular IOD using the Volume Frame of Reference may constrain the alignment of frames with respect to the axes of the Volume Frame of Reference. For example, Figure C.8.24.2-1 illustrates the use of the Volume Frame of Reference with frames whose rows are parallel to the X_V axis and columns are parallel to the Y_V axis and whose origins lie on the Z_V axis.

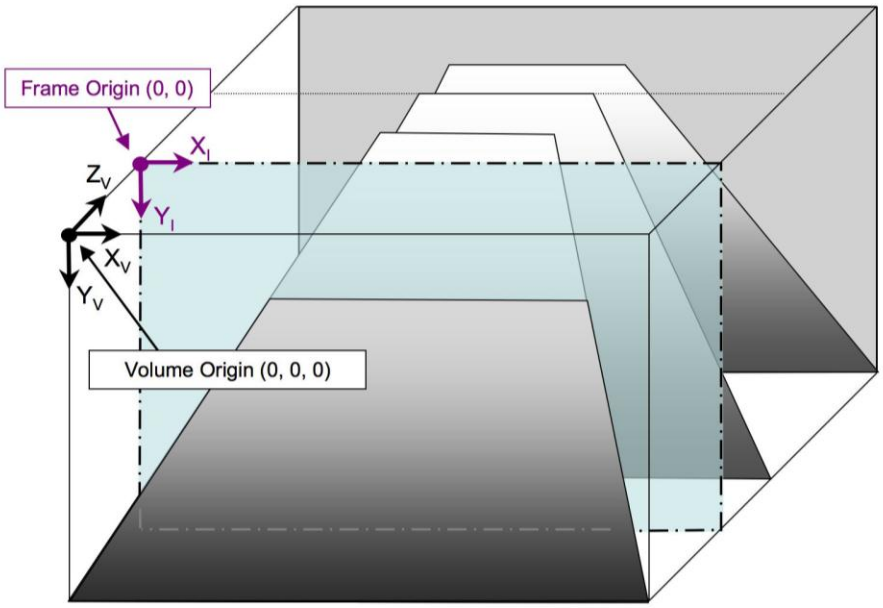

Figure C.8.24.2-1: Volume Frame of Reference

C.8.24.2.1.2 Transducer Frame of Reference

The Transducer Frame of Reference is a Right-hand Coordinate System consisting X_X, Y_X, and Z_X axes originating at a reference "Transducer Origin" defined as the geometric center of the transducer face.

The orientation of the Transducer Frame of Reference relative to the Volume Origin is such that the Y_X axis is normal to the transducer face and the "direction reference" (*i.e.* transducer tactile marker or zero reference) is aligned with the positive X_X axis. A transformation is specified between the Volume Frame of Reference and the Transducer Frame of Reference to define the position of the transducer relative to the volume. This transformation is specified by the Volume to Transducer Mapping Matrix (0020,9309).

The Transducer Frame of Reference recognizes two types of scan geometry: 1) a scan geometry with a real apex such as would be the case for a pyramid, toroid or rotational volume acquisition, or 2) a scan geometry for which there is no specific apex. The point (x_A, y_A, z_A) is the apex (or phase center) of the acquisition volume geometry in the Volume Frame of Reference. The apex (x_A, y_A, z_A) may be located in the volume or exterior to it.

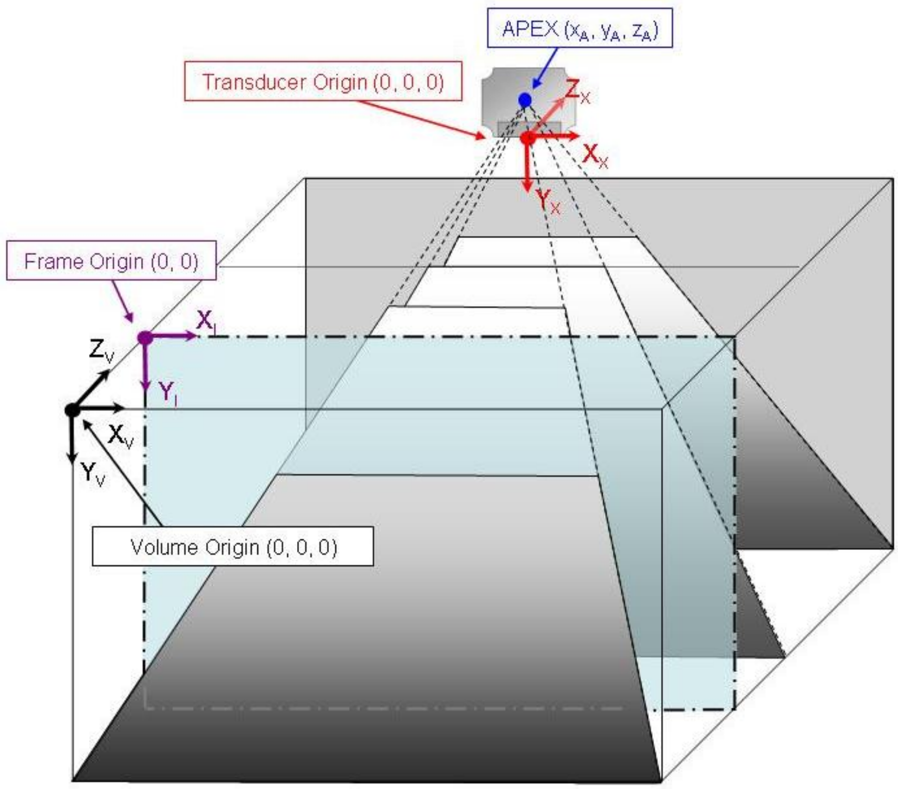

Figure C.8.24.2-2: Transducer Frame of Reference

C.8.24.2.1.3 Table Frame of Reference

There also may exist a fixed equipment reference called the Table Frame of Reference, a Right-hand Coordinate System consisting of X_T, Y_T, and Z_T axes originating at a reference “Table Origin”. See Figure C.8.24.2-3.

Note: In this context the Table Frame of Reference refers to a fixed coordinate system in space that may be provided by a variety of source devices such as coordinates from a magnetic position sensor, LED sensor array, a physical scanner gantry, or similar device.

A transformation may be specified between the Volume Frame of Reference and the Table Frame of Reference to define the position and orientation of the volume relative to this external frame of reference. This transformation is specified by the Volume to Table Mapping Matrix (0020,930A).

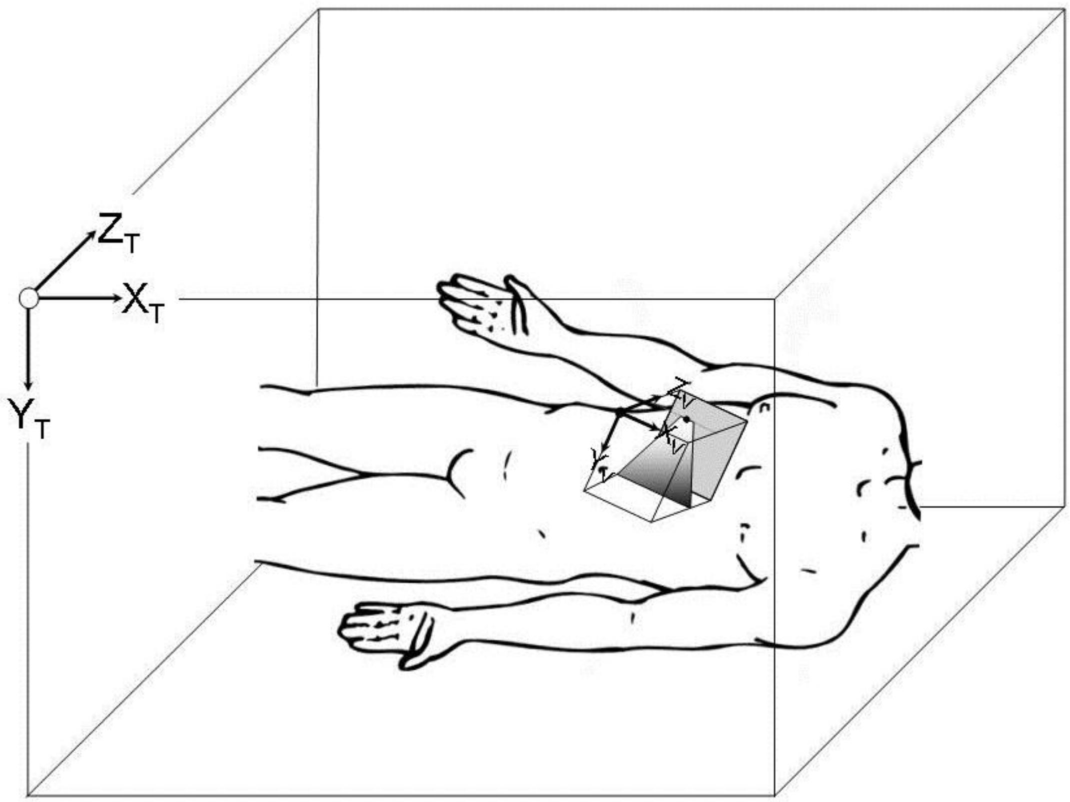

Figure C.8.24.2-3: Table Frame of Reference

C.8.24.2.2 Ultrasound Frame of Reference Module Attributes

C.8.24.2.2.1 Volume to Transducer Mapping Matrix

The Volume to Transducer Mapping Matrix (0020,9309) is used to describe the relationship between the Transducer Frame of Reference coordinate system and the Volume Frame of Reference coordinate system.

The Volume to Transducer Mapping Matrix ($[M_{TV}]=[P]*[Q]$) describes how to transform a point (X_V, Y_V, Z_V) in the Volume coordinate system into (X_X, Y_X, Z_X) in the Transducer coordinate system according to the equation below.

$$\begin{bmatrix} X_X \\ Y_X \\ Z_X \\ 1 \end{bmatrix} = \begin{bmatrix} P_{XX} & P_{YX} & P_{ZX} & Q_X \\ P_{XY} & P_{YY} & P_{ZY} & Q_Y \\ P_{XZ} & P_{YZ} & P_{ZZ} & Q_Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_V \\ Y_V \\ Z_V \\ 1 \end{bmatrix}$$

Where:

$X_V Y_V Z_V$ The voxel location (in mm) in the Volume Frame of Reference
$X_X Y_X Z_X$ The voxel location (in mm) in the Transducer Frame of Reference
P_{ij}, P_{ij}, P_{ij} A 3x3 matrix of direction cosine values as measured to the Transducer origin from the volume origin.

Q_x, Q_y, Q_z The translation values (in mm) describe the location in mm of the Transducer Frame of Reference (X_X, Y_X, Z_X) origin from the Volume Reference Origin (X_V, Y_V, Z_V)measured in millimeters along the Volume axes i.e. to the transducer origin from the volume origin.

C.8.24.2.2.2 Volume to Table Mapping Matrix

The Volume to Table Mapping Matrix (0020,930A) is used to describe the relationship between the Volume Frame of Reference coordinate system and a modality specific equipment coordinate system. This mapping can be used only with systems that have a well-defined equipment coordinate system.

The Volume to Table Mapping Matrix ($[M_{VG}]=[R]*[S]$) describes how to transform a point ($X_V Y_V Z_V$) in the Volume coordinate system into (X_T, Y_T, Z_T) in the Table coordinate system according to the equation below.

$$\begin{bmatrix} X_T \\ Y_T \\ Z_T \\ 1 \end{bmatrix} = \begin{bmatrix} R_{XX} & R_{YX} & R_{ZX} & S_X \\ R_{XY} & R_{YY} & R_{ZY} & S_Y \\ R_{XZ} & R_{YZ} & R_{ZZ} & S_Z \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_V \\ Y_V \\ Z_V \\ 1 \end{bmatrix}$$

Where:

$X_V Y_V Z_V$ The voxel location (in mm) in the Volume Frame of Reference

$X_T Y_T Z_T$ The voxel location (in mm) in the Table Frame of Reference

R_{ij}, R_{ij}, R_{ij} A 3x3 matrix of direction cosine values as measured to the gantry origin from the volume origin.

S_x, S_y, S_z The translation values (in mm) describe the location in mm of the Table Frame of Reference (X_T, Y_T, Z_T) origin from the Volume Reference Origin (X_V, Y_V, Z_V)measured in millimeters along the table axes I.e. to the table origin from the volume origin.

Note: The Mapping Matrices are rigid transformations that involve only translations and rotations. Mathematically, the matrix is orthonormal and describes six degrees of freedom: three translations, and three rotations.

Amend PS 3.17 Annex Registration (Informative)

O.5 MATRIX REGISTRATION

A 4x4 ~~**homogeneous**~~ **affine** transformation matrix describes spatial rotation, translation, scale changes and affine transformations that register referenced images to the Registration IE's **homogeneous** RCS. These steps are expressible in a single matrix, or as a sequence of multiple independent rotations, translations, or scaling, each expressed in a separate matrix. Normally, registrations are rigid body, involving only rotation and translation. Changes in scale or affine transformations occur in atlas registration or to correct minor mismatches.

Amend PS 3.17 Annex P Transforms and Mappings (Informative)

The ~~**Homogenous**~~ **Affine** Transform Matrix is of the following form.

$$\begin{bmatrix} M_{11} & M_{12} & M_{13} & T_x \\ M_{21} & M_{22} & M_{23} & T_y \\ M_{31} & M_{32} & M_{33} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

This matrix requires the bottom row to be [0 0 0 1] **to preserve the homogeneous coordinates**.

The matrix can be of type: RIGID, RIGID_SCALE and AFFINE. These different types represent different conditions on the allowable values for the matrix elements.

RIGID:

This transform requires the matrix obey orthonormal transformation properties:

$$\sum_{i=1}^{3} M_{ij} M_{ik} = \delta_{jk}$$ for all combinations of j = 1,2,3 and k = 1,2,3 where delta = 1 for i = j and zero otherwise.

The expansion into non-matrix equations is:

$M_{11} M_{11} + M_{21} M_{21} + M_{31} M_{31} = 1$ where j = 1, k = 1

$M_{11} M_{12} + M_{21} M_{22} + M_{31} M_{32} = 0$ where j = 1, k = 2

$M_{11} M_{13} + M_{21} M_{23} + M_{31} M_{33} = 0$ where j = 1, k = 3

$M_{12} M_{11} + M_{22} M_{21} + M_{32} M_{31} = 0$ where j = 2, k = 1

$M_{12} M_{12} + M_{22} M_{22} + M_{32} M_{32} = 1$ where j = 2, k = 2

$M_{12} M_{13} + M_{22} M_{23} + M_{32} M_{33} = 0$ where j = 2, k = 3

$M_{13} M_{11} + M_{23} M_{21} + M_{33} M_{31} = 0$ where j = 3, k = 1

$M_{13} M_{12} + M_{23} M_{22} + M_{33} M_{32} = 0$ where j = 3, k = 2

$M_{13} M_{13} + M_{23} M_{23} + M_{33} M_{33} = 1$ where j = 3, k = 3

The Frame of Reference Transformation Matrix ${}^{A}M_{B}$ describes how to transform a point (${}^{B}x$,${}^{B}y$,${}^{B}z$) with respect to RCS_B into (${}^{A}x$,${}^{A}y$,${}^{A}z$) with respect to RCS_A.

$$\begin{bmatrix} {}^{A}X \\ {}^{A}Y \\ {}^{A}Z \\ 1 \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & T_1 \\ M_{21} & M_{22} & M_{23} & T_2 \\ M_{31} & M_{32} & M_{33} & T_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} {}^{B}X \\ {}^{B}Y \\ {}^{B}Z \\ 1 \end{bmatrix}$$

The matrix above consists of two parts: a rotation and translation as shown below;

$$\text{Rotation: } \begin{bmatrix} M_{11} & M_{12} & M_{13} & 0 \\ M_{21} & M_{22} & M_{23} & 0 \\ M_{31} & M_{32} & M_{33} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \text{Translation: } \begin{bmatrix} 1 & 0 & 0 & T_1 \\ 0 & 1 & 0 & T_2 \\ 0 & 0 & 1 & T_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The first column [M_{11},M_{21},M_{31}] are the direction cosines (projection) of the X-axis of RCS_B with respect to RCS_A. The second column [M_{12},M_{22},M_{32}] are the direction cosines (projection) of the Y-axis of RCS_B with respect to RCS_A. The third column [M_{13},M_{23},M_{33}] are the direction cosines (projection) of the Z-axis of RCS_B with respect to RCS_A. The fourth column [T_1,T_2,T_3] is the origin of $RCS_B$with respect to RCS_A.

There are three degrees of freedom representing rotation, and three degrees of freedom representing translation, giving a total of six degrees of freedom.

RIGID_SCALE

The following constraint applies:

$$\sum_{i=1}^{3} M_{ij} M_{ik} = \delta_{jik} S_j^2$$ for all combinations of j = 1,2,3 and k = 1,2,3 where delta = 1 for i=j and zero otherwise.

The expansion into non-matrix equations is:

$M_{11}\, M_{11} + M_{21}\, M_{21} + M_{31}\, M_{31} = S_1^2$ where j = 1, k = 1

$M_{11}\, M_{12} + M_{21}\, M_{22} + M_{31}\, M_{32} = 0$ where j = 1, k = 2

$M_{11}\, M_{13} + M_{21}\, M_{23} + M_{31}\, M_{33} = 0$ where j = 1, k = 3

$M_{12}\, M_{11} + M_{22}\, M_{21} + M_{32}\, M_{31} = 0$ where j = 2, k = 1

$M_{12}\, M_{12} + M_{22}\, M_{22} + M_{32}\, M_{32} = S_2^2$ where j = 2, k = 2

$M_{12}\, M_{13} + M_{22}\, M_{23} + M_{32}\, M_{33} = 0$ where j = 2, k = 3

$M_{13}\, M_{11} + M_{23}\, M_{21} + M_{33}\, M_{31} = 0$ where j = 3, k = 1

$M_{13}\, M_{12} + M_{23}\, M_{22} + M_{33}\, M_{32} = 0$ where j = 3, k = 2

$M_{13}\, M_{13} + M_{23}\, M_{23} + M_{33}\, M_{33} = S_3^2$ where j = 3, k = 3

The above equations show a simple way of extracting the spatial scaling parameters Sj from a given matrix. The units of S_j^2 is the RCS unit dimension of one millimeter.

This type can be considered a simple extension of the type RIGID. The RIGID_SCALE is easily created by pre-multiplying a RIGID matrix by a diagonal scaling matrix as follows:

$$M_{RBWS} = \begin{bmatrix} S_1 & 0 & 0 & 0 \\ 0 & S_2 & 0 & 0 \\ 0 & 0 & S_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * M_{RB}$$

where M_{RBWS} is a matrix of type RIGID_SCALE and M_{RB} is a matrix of type RIGID.

AFFINE:

No constraints apply to this matrix, so it contains twelve degrees of freedom. This type of Frame of Reference Transformation Matrix allows shearing in addition to rotation, translation and scaling.

For a RIGID type of Frame of Reference Transformation Matrix, the inverse is easily computed using the following formula (inverse of an orthonormal matrix):

annex. $$({}^{A}\mathbf{M}_{B})^{-1} = \begin{bmatrix} M_{11} & M_{12} & M_{13} & T_x \\ M_{21} & M_{22} & M_{23} & T_y \\ M_{31} & M_{32} & M_{33} & T_z \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} M_{11} & M_{21} & M_{31} & M_{11}T_x + M_{21}T_y + M_{31}T_z \\ M_{12} & M_{22} & M_{32} & M_{12}T_x + M_{22}T_y + M_{32}T_z \\ M_{13} & M_{23} & M_{33} & M_{13}T_x + M_{23}T_y + M_{33}T_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

For RIGID_SCALE and AFFINE types of Registration Matrices, the inverse cannot be calculated using the above equation, and must be calculated using a conventional matrix inverse operation.